Stebėtoją ir stebinį skiriančios kombinatorinės interpretacijos

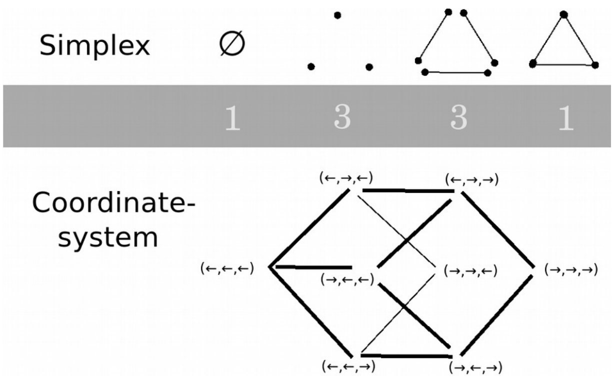
Combinatorial Interpretations Which Distinguish Observer and Observed

Andrius Jonas Kulikauskas
UCSD (1986-1993), Vytauto g. 15, Eičiūnų km, LT-64424 Alytaus raj
ms@ms.lt

The measurement problem, whether and how the wave function collapses, persists as a challenge for attempts to comprehend quantum mechanics.

Algebraic combinatorics is relevant in that it shows how different interpretations of the same system can exhibit different symmetries.

The binomial theorem grounds four conceptual frameworks. Consider the row 1 3 3 1 in Pascal's triangle. We may think of it as counting the subsets of the vertices of a triangle (a 2-simplex).



1pav. Simplex vs. Coordinate-system

But the numbers also count the points establishing a coordinate-system. Here we have a symmetric choice between "choose this" ← and "choose not this" →. It is semantic. Whereas with the simplex we have an asymmetric choice between "choose" ↔ and "do not choose" ∅. Each number counts a different kind of sub-simplex. Choosing no vertices (the center) and choosing all vertices (the totality) are two syntactic extremes.

A variant of Pascal's triangle counts the sub-polytopes of cross-polytopes. An octahedron is constructed from axes linking pairs of vertices, like particles and anti-particles. It has $1 = 1 \times 2^0$ center, $6 = 3 \times 2^1$ vertices, $12 = 3 \times 2^2$ edges, $8 = 1 \times 2^3$ faces and (combinatorially) no volume, no totality!

A hypercube is a cross-polytope's dual. It halves space in various dimensions. A cube has 1 3-D whole, 6 2-D faces, 12 1-D edges, 8 0-D vertices, and no center!

Simplexes $\prod_i(\varnothing + \leftrightarrow_i)$, coordinate-systems $\prod_i(\leftarrow_i + \rightarrow_i)$, cross-polytopes $\prod_i(\varnothing + \leftarrow_i + \rightarrow_i)$ and hypercubes $\prod_i(\leftrightarrow_i + \leftarrow_i + \rightarrow_i)$ all express the binomial theorem. However, they differ in their symmetries. Cognitively, this is because we distinguish between negative qualities, $\rightarrow_i \neq \rightarrow_j$, whereas we conflate all nonexistences $\varnothing_i = \varnothing_j = \varnothing$.

The simplex's symmetry group is the symmetric group $|S_n| = n!$, the cross-polytope's and hypercube's is the hyperoctahedral group $|B_n| = n!2^n$ and the coordinate-system's is the subgroup $|D_n| = n!2^{n-1}$.

Physically, these frameworks model distinctions between an observer and an observed. Classically, objectively, there are never gaps in nature where nothing happens, but merely changes in state between "this" and "that", as with the coordinate-systems. Here time has no intrinsic direction. An observer, however, can subjectively carve up time as they like, in which case an event may be observed or not, just as a simplex's vertex may be chosen or not. Time then has a clear direction in which new choices are continuously added. We can distinguish forwards and backwards.

The symmetry groups of the frameworks are the Weyl groups of the root systems of the classical Lie algebras, which offer more combinatorial insights. [1] The root system A_n with simple roots $x_n - x_{n-1}, \ldots, x_2 - x_1$ can be read as maintaining the duality of forwards and backwards by imposing a complex norm upon the corresponding Lie group. That duality can be collapsed in two ways which reduce the complex norm to a real norm. B_n appeals to an external zero in absolute reality to glue together the two directions in time. D_n fuses an internal zero, defining a relative reality, as with the coordinate systems. C_n folds the duality, doubling it, establishing a quaternion norm, as when position and momentum are coupled.

Such a reading of the combinatorics suggests that the complex numbers are most fundamental. They maintain that duality by which observers distinguish forwards and backwards. We, subjective observers, operate in the same framework as an uncollapsed wave function. Interacting with quantum processes, we circumscribe and construct a classical, objective reality. Wave functions collapse contingently, in that reality. A collapse could be undone by constructing a different reality. Observers, as such, are not in classical reality.

Reikšminiai žodžiai: matavimo problema, simetrija, binomo formulė, Lie algebros.

Literatūra

[1] A.J. Kulikauskas, *Keturių klasikinių Lie grupių ir algebrų kombinatorinės ištakos*, Lietuvos matematikų draugijos LX konferencija, (Vilnius, 2019).